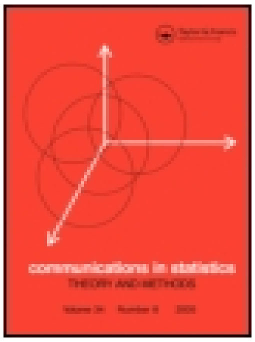

Communications in Statistics - Theory and Methods

ISSN: 0361-0926 (Print) 1532-415X (Online) Journal homepage: http://www.tandfonline.com/loi/lsta20

Slashed generalized exponential distribution

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Abstract

In this paper, we introduce an extension of the generalized exponential distribution, making it more robust against possible influential observations. The new model is defined as the quotient between a generalized exponential random variable and a beta distributed random variable with one unknown parameter. The resulting distribution is a distribution with greater kurtosis than the generalized exponential distribution. Probability properties of the distribution such as moments and asymmetry and kurtosis are studied. Likewise, statistical properties are investigated using the method of moments and the maximum likelihood approach. Two real data analyses are reported illustrating better performance of the new model over the generalized exponential model.

KEY WORDS: Generalized exponential distribution, kurtosis, maximum likelihood, slash distribution.

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1 Introduction

The ordinary slash distribution is closely related to the normal distribution. Its stochastic representation is the quotient between two independent random variables, a normal one in the numerator and the power of a uniform distributed random variable in the denominator. More specifically, it is said that a random variable S follows a standard slash distribution with shape parameter q if

$$S = Z/U^{\frac{1}{q}}, \tag{1}$$

where $Z \sim N(0, 1)$, $U \sim U(0, 1)$, Z is independent of U and $q > 0$ (see Johnson, Kotz and Balakrishnan 1995). This distribution presents heavier tails than the normal distribution, that is, more kurtosis. Properties of this distribution are discussed in Rogers and Tukey (1972) and Mosteller and Tukey (1977). Maximum likelihood estimation for a location-scale extension of the standard slash distribution is considered in Kadafar (1982). Asymmetric and symmetric multivariate versions were considered in Wang and Genton (2006). Gómez et al. (2007) and Gómez and Venegas (2008) extend the ordinary slash distribution by replacing the normal distribution in the numerator by the family of univariate and multivariate elliptical distributions. Several extensions of distributions with positive support using such methodology were considered in Gómez et al. (2009), Olivares-Pacheco et. al. (2010), Olmos et al. (2012, 2014) and Iriarte et al. (2015).

On the other hand, the generalized exponential distribution was studied in Gupta and Kundu (1999), which is a particular case of the exponentiated Weibull distribution, with zero location, introduced by Mudholkar et al. (1995). Thus, a random variable X follows the generalized exponential distribution with scale parameter λ and shape parameter α if its density function is given by

$$f(x; \lambda, \alpha) = \alpha\lambda \left(1 - e^{-\lambda x}\right)^{\alpha-1} e^{-\lambda x} I\{x > 0\}, \tag{2}$$

with $\lambda > 0$ and $\alpha > 0$, hereafter denoted $X \sim GE(\lambda, \alpha)$. More recently, this distribution has been used in different areas by several authors including Gupta and Kundu (2001a, 2001b, 2002, 2007), Mitra and Kundu (2008) and Kundu and Gupta (2008), among others.

The main object of this note is to introduce another extension of the generalized exponential distribution by "slashing" it, that is by using the slash idea for generating new positive distributions, with possibly greater kurtosis.

The paper is organized as follows. Section 2 deals with the stochastic representation for the new distribution and studies its moments, and asymmetry and kurtosis coefficients. In Section 3 we develop moments and maximum likelihood estimation. In Section 4, we perform a small scale simulation study of the maximum likelihood estimators for parameters of the new distribution, the main conclusion is that the approach yields good parameter recovery. Section 5 is dedicated to present an analysis of two real data sets illustrating the performance of the proposed methodology. Final remarks and conclusions are deferred to Section 6.

2 Density function and properties

In this section we introduce the stochastic representation, density function, properties and plots of the new distribution.

2.1 Stochastic representation

The stochastic representation for the new distribution is given by

$$Z = \frac{X}{Y}, \tag{3}$$

where $X \sim GE(\lambda, \alpha)$ and $Y \sim Beta(q, 1)$ are independent, $\lambda > 0$, $\alpha > 0$, $q > 0$ and the distribution of Z, hereafter called the slashed generalized exponential distribution. We denote this by $Z \sim SGE(\lambda, \alpha, q)$.

2.2 Density function

The following proposition reveals the pdf for the SGE distribution, which is generated using the representation given in (3).

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Proposition 1 *Let $Z \sim SGE(\lambda, \alpha, q)$. Then, the pdf of Z is given by*

$$f_Z(z; \lambda, \alpha, q) = \frac{\alpha q}{\lambda^q z^{q+1}} J_{(\alpha,q)} \left(1 - e^{-\lambda z}\right), \tag{4}$$

where $\lambda > 0, \alpha > 0, q > 0, z > 0$ and $J_{(\alpha,q)}(t) = \int_0^t log^q \left(\frac{1}{1-u}\right) u^{\alpha-1} du.$

Proof. Using the representation given in (3) and computing the Jacobian of the required transformation, we have that

$$\left.\begin{array}{l} Z = \frac{X}{Y} \\ W = Y \end{array}\right\} \Rightarrow \left.\begin{array}{l} X = ZW \\ Y = W \end{array}\right\} \Rightarrow J = \begin{vmatrix} \frac{\partial X}{\partial Z} & \frac{\partial X}{\partial W} \\ \frac{\partial Y}{\partial Z} & \frac{\partial Y}{\partial W} \end{vmatrix} = \begin{vmatrix} w & z \\ 0 & 1 \end{vmatrix} = w$$

$$\begin{aligned} f_{Z,W}(z, w) &= |J| f_{X,Y}(zw, w) \\ f_{Z,W}(z, w) &= w f_X(zw) f_Y(w)\, ,\ 0 < w < 1\, ,\ z > 0, \end{aligned}$$

so that marginalizing with respect to the random variable W we obtain the density function corresponding to the random variable Z, namely

$$f_Z(z; \lambda, \alpha, q) = \lambda\alpha q \int_0^1 w^q (1 - e^{-\lambda zw})^{\alpha-1} e^{-\lambda zw} dw.$$

Finally, making the variable transformation $u = 1 - e^{-\lambda zw}$ and working inside the integral the result follows □.

Particularly, if $\lambda = \alpha = q = 1$, the canonic slashed generalized exponential distribution follows, denoted by $Z \sim SGE(1, 1, 1)$. Then, the density function of Z is given by

$$f_Z(z) = \frac{1}{z^2}(1 - e^{-z} - ze^{-z})\, , \quad z > 0. \tag{5}$$

2.3 Probabilistic properties

In this subsection, we study basic properties of the SGE distribution. Let $Z \sim SGE(\lambda, \alpha, q)$, so that

1. $\lim_{q\to\infty} f_Z(z; \lambda, \alpha, q) = \alpha\lambda \left(1 - e^{-\lambda x}\right)^{\alpha-1} e^{-\lambda x} I\{x > 0\};$

2. $\lim_{q\to 0} f_Z(z; \lambda, \alpha, q) = 0.$

Another important property is stated and proved next.

Proposition 2 *Let* $Y|U = u \sim GE(u^{1/q}\lambda, \alpha)$ *and* $U \sim U(0,1)$ *then* $Y \sim SGE(\lambda, \alpha, q)$.

Proof. We can write

$$f_Y(y;\lambda,\alpha,q) = \int_0^1 f_{Y|U}(y|u) f_U(u) du = \int_0^1 \alpha u^{1/q}\lambda \left(1 - e^{-u^{1/q}\lambda y}\right)^{\alpha-1} e^{-u^{1/q}\lambda y} du$$

The result follows then by making the variable change $t = 1 - e^{-u^{1/q}\lambda y}$ □.

Remark 1 *Property 1 reveals that this distribution has as a special limiting case the generalized exponential distribution. On the other hand, Proposition 2 reveals that the distribution is a scale mixture of the generalized exponential distribution and the uniform distribution in the* $(0,1)$ *interval.*

2.4 Moments

We present a general formula for the r-moment of the SGE distribution.

Proposition 3 *Let* $Z \sim SGE(\lambda, \alpha, q)$*. Then, for* $r = 1, 2, ...$ *and* $q > r$ *it follows that the* r*-th moment of* Z *can be written as*

$$\mu_r = E(Z^r) = \frac{\alpha q \Gamma(r+1)}{(q-r)\lambda^r} \sum_{i=0}^{\infty} (-1)^i \frac{c(\alpha-1,i)}{(i+1)^{r+1}}, \tag{6}$$

where $c(\alpha-1,i) = \frac{(\alpha-1)\times\ldots\times(\alpha-i)}{i!}$.

Proof. Using the stochastic representation for the distribution given in (3), we have that

$$\mu_r = E(Z^r) = E\left(\left(\frac{X}{Y}\right)^r\right) = E(X^r Y^{-r}) = E(X^r)\,E(Y^{-r}),$$

where $E(Y^{-r}) = \frac{q}{q-r}$, $q > r$ and $E(X^r) = \frac{\alpha\Gamma(r+1)}{\lambda^r}\sum_{i=0}^{\infty}(-1)^i\frac{c(\alpha-1,i)}{(i+1)^{r+1}}$ are the moments of the $GE(\lambda,\alpha)$ distribution □.

Proposition 4 *If $Y \sim SGE(\lambda, \alpha, q)$, then the moments generating function for the random variable Y is given by*

$$M_Y(t) = \frac{q}{\lambda^q} \int_0^{\lambda} \frac{\Gamma(\alpha+1)\Gamma(1-\frac{t}{w})}{\Gamma(\alpha - \frac{t}{w} + 1)} w^{q-1} dw. \quad (7)$$

Proof. Using Proposition 2, we can write

$$M_Y(t) = E(e^{tY}) = E(E(e^{tY}|U)) = E\left(\frac{\Gamma(\alpha+1)\Gamma(1-\frac{t}{U^{1/q}\lambda})}{\Gamma(\alpha - \frac{t}{U^{1/q}\lambda} + 1)}\right) = \int_0^1 \frac{\Gamma(\alpha+1)\Gamma(1-\frac{t}{u^{1/q}\lambda})}{\Gamma(\alpha - \frac{t}{u^{1/q}\lambda} + 1)} du$$

and hence the result follows making the variable transformation $w = u^{1/q}\lambda$ □.

Corollary 1 *If $Z \sim SGE(\lambda, \alpha, q)$ then, it follows that*

1. $\mu_1 = E(Z) = \frac{q(d_1)}{\lambda(q-1)}$ $, q > 1;$
2. $\mu_2 = E(Z^2) = \frac{q(d_2+d_1^2)}{\lambda^2(q-2)}$ $, q > 2;$
3. $\mu_3 = E(Z^3) = \frac{q[d_3+3d_1d_2+d_1^3]}{\lambda^3(q-3)}$ $, q > 3;$
4. $\mu_4 = E(Z^4) = \frac{q[d_4+3d_2^2+4d_1d_3+6d_1^2d_2+(d_1)^4]}{\lambda^4(q-4)}$ $, q > 4;$
5. $Var(Z) = \frac{q}{\lambda^2}\left[\frac{(d_2+d_1^2)}{q-2} - \frac{qd_1^2}{(q-1)^2}\right]$ $, q > 2,$

where

$d_1 = \psi(\alpha+1) - \psi(1)$, $d_2 = \psi'(1) - \psi'(\alpha+1)$, $d_3 = \psi''(\alpha+1) - \psi''(1)$ $d_4 = \psi'''(1) - \psi'''(\alpha+1)$ *and $\psi^m(x)$, is the polygamma function of order m.*

Corollary 2 *Let $Z \sim SGE(\lambda, \alpha, q)$, then the asymmetry coefficient, ($\sqrt{\beta_1}$) and the kurtosis coefficient (β_2) for $q > 3$ and $q > 4$ are, respectively,*

$$\sqrt{\beta_1} = \frac{A1}{\sqrt{q}(q-3)[(q-1)^2(d_2+d_1^2) - q(q-2)d_1^2]^{\frac{3}{2}}}, \quad q > 3,$$

where

$A1 = \sqrt{(q-2)}[(q-1)^3(q-2)(d_3+3d_1d_2+d_1^3) - 3q(q-1)^2(q-3)(d_1d_2+d_1^3) + 2q^2(q-2)(q-3)d_1^3].$

$$\beta_2 = \frac{C1}{(q-3)(q-4)[q(q-1)^4(d_2+d_1^2)^2 - 2q^2(q-1)^2(q-2)(d_1^2 d_2 + d_1^4) + q^3(q-2)^2 d_1^4]}, \ q > 4,$$

where

$C1 = (q-2)[(q-1)^4(q-2)(q-3)(d_4 + 3d_2^2 + 4d_1d_3 + 6d_1^2 d_2 + d_1^4) - 4q(q-1)^3(q-2)(q-4)(d_1d_3 + 3d_1^2 d_2 + d_1^4) + 6q^2(q-1)^2(q-3)(q-4)(d_1^2 d_2 + d_1^4) - 3q^3(q-2)(q-3)(q-4)d_1^4].$

Remark 2 *The asymmetry and kurtosis coefficients were obtained using:*

$$\sqrt{\beta_1} = \frac{\mu_3 - 3\mu_1\mu_2 + 2\mu_1^3}{(\mu_2 - \mu_1^2)^{\frac{3}{2}}} \quad and \quad \beta_2 = \frac{\mu_4 - 4\mu_1\mu_3 + 6\mu_2\mu_1^2 - 3\mu_1^4}{(\mu_2 - \mu_1^2)^2}.$$

Remark 3 *Figure 2 and 3 depicts plots for the asymmetry and kurtosis coefficients of the distribution $GE(\lambda, \alpha)$ and $SGE(\lambda, \alpha, q)$ respectively. Notice that asymmetry and kurtosis coefficients are larger in SGE model that in GE model, and these ones no depend of λ because λ is a scale parameter.*

3 Inference

3.1 Moment estimators

Rewriting the first moment with λ isolated and replacing E(Z) by the sample mean $\overline{Z}$, we have the equation

$$\lambda = \frac{q(d_1)}{\overline{Z}(q-1)}. \tag{8}$$

Therefore, using (8) and replacing the second and third population moments by the corresponding second and third sampling moments we obtain the following equations:

$$\overline{Z^2} = \frac{(d_2 + d_1^2)(q-1)^2\overline{Z}^2}{q d_1^2 (q-2)}; \tag{9}$$

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$$\overline{Z^3} = \frac{(d_3 + 3d_1 d_2 + d_1^3)(q-1)^3 \overline{Z}^3}{q^2 d_1^3 (q-3)}. \tag{10}$$

The system of equations generated by (9) and (10) needs to be solved numerically using `MAPLE`, for example, leading to the estimators $\widehat{\alpha}$ and $\widehat{q}$. The estimator $\widehat{\lambda}$, is obtained from equation (8), replacing q by $\widehat{q}$ and computing d_1 by using estimator $\widehat{\alpha}$.

3.2 Maximum likelihood estimators

In this section, it is presented the maximum likelihood equations for parameters (λ, α, q) of the SGE model. Being $Z_1, Z_2, \ldots, Z_n$ a random sample from random variable $Z \sim SGE(\lambda, \alpha, q)$, the log-likelihood function can be expressed as

$$\log L(\lambda, \alpha, q) = c(\lambda, \alpha, q) - (q+1)\sum_{i=1}^{n} \log(z_i) + \sum_{i=1}^{n} \log(J_{(\alpha,q)}\left(1 - e^{-\lambda z_i}\right)), \tag{11}$$

where $c(\lambda, \alpha, q) = -nq\log(\lambda) + n\log(\alpha) + n\log(q)$.

The maximum likelihood estimators are obtained by maximizing the log-likelihood function given in (11). Deriving the log-likelihood function with respect to each parameter, the following estimating equations are obtained:

$$\sum_{i=1}^{n} \frac{J_1(z_i)}{J(z_i)} = \frac{nq}{\lambda}; \tag{12}$$

$$\sum_{i=1}^{n} \frac{J_2(z_i)}{J(z_i)} = \frac{-n}{\alpha}; \tag{13}$$

$$\sum_{i=1}^{n} \frac{J_3(z_i)}{J(z_i)} = nlog(\lambda) - \frac{n}{q} + \sum_{i=1}^{n} log(z_i); \tag{14}$$

where $J(z_i) = J_{(\alpha,q)}\left(1 - e^{-\lambda z_i}\right)$ is defined in (4), $J_1(z_i) = \frac{\partial J(z_i)}{\partial \lambda}$, $J_2(z_i) = \frac{\partial J(z_i)}{\partial \alpha}$ and $J_3(z_i) = \frac{\partial J(z_i)}{\partial q}$. Equations (12),(13) and (14) must be solved using numerical procedures as, for example, the function `optim` in software `R`.

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Given the complexity of the likelihood function, it is possible to work with the observed information matrix to compute estimates of the large sample variances for parameter estimates. This matrix is obtained by computing the Hessian matrix and evaluating it at the maximum likelihood estimates. It follows after lengthy algebraic manipulations that the Hessian matrix is given by

$$H_n(\lambda, \alpha, q) = \begin{pmatrix} \frac{nq}{\lambda^2} + \sum_{i=1}^{n} \frac{d}{d\lambda} \frac{J_1(z_i)}{J(z_i)} & \sum_{i=1}^{n} \frac{d}{d\alpha} \frac{J_1(z_i)}{J(z_i)} & \frac{-n}{\lambda} + \sum_{i=1}^{n} \frac{d}{dq} \frac{J_1(z_i)}{J(z_i)} \\ \sum_{i=1}^{n} \frac{d}{d\lambda} \frac{J_2(z_i)}{J(z_i)} & \frac{-n}{\alpha^2} + \sum_{i=1}^{n} \frac{d}{d\alpha} \frac{J_2(z_i)}{J(z_i)} & \sum_{i=1}^{n} \frac{d}{dq} \frac{J_2(z_i)}{J(z_i)} \\ \frac{-n}{\lambda} + \sum_{i=1}^{n} \frac{d}{d\lambda} \frac{J_3(z_i)}{J(z_i)} & \sum_{i=1}^{n} \frac{d}{d\alpha} \frac{J_3(z_i)}{J(z_i)} & \frac{-n}{q^2} + \sum_{i=1}^{n} \frac{d}{dq} \frac{J_3(z_i)}{J(z_i)} \end{pmatrix}.$$

The observed information matrix is given by $H_n(\hat{\lambda}, \hat{\alpha}, \hat{q})$.

4 Simulation study

In this section a simulation study is conducted aiming at investigating maximum likelihood estimation performance for parameters λ, α and q under the SGE model. Using Algorithm 1 (below), 1000 random samples of sizes $n = 50$, 100 and 200 were generated under the SGE model with different parameter values. A summary of the results from the study are depicted in Table 2. Empirical means corresponds the estimated parameters means over the 1000 simulated samples and SD the empirical standard deviation over the 1000 simulated samples. One of the main conclusions is that as sample size increase, estimates become closer to the true parameter values. Further, results indicate that estimated standard errors become smaller as sample size increases.

We present next Algorithm 1, used to generate samples from $Z \sim SGE(\lambda, \alpha, q)$.

Algorithm 1

1. Generate $U \sim U(0, 1)$;
2. Compute $X = \frac{-log(1-U^{\frac{1}{\alpha}})}{\lambda}$;
3. Generate $Y \sim U(0, 1)$;
4. Compute $Z = \frac{X}{Y^{\frac{1}{q}}}$,

5 Real Data Illustration

In this section, we present two illustrations using real data sets aiming at comparing in terms of model fitting the SGE and GE (generalized exponential) models. Comparisons are made using the likelihood approach, namely the likelihood ratio statistics and AIC and BIC type criteria.

5.1 Illustration 1

The first data set was previously analysed in Chhikara and Folks (1977). It corresponds to the 46 active repair times (in hours) for an airborne communication transceiver with the following observed values: 0.2, 0.3, 0.5, 0.5, 0.5, 0.5, 0.6, 0.6, 0.7, 0.7, 0.7, 0.8, 0.8, 1.0, 1.0, 1.0, 1.0, 1.1, 1.3, 1.5, 1.5, 1.5, 1.5, 2.0, 2.0, 2.2, 2.5, 2.7, 3.0, 3.0, 3.3, 3.3, 4.0, 4.0, 4.5, 4.7, 5.0, 5.4, 5.4, 7.0, 7.5, 8.8, 9.0, 10.3, 22.0, 24.5.

Table 3 presents summary statistics for the above data set where b_1 and b_2 correspond to the asymmetry and kurtosis coefficients, respectively. We call attention to the fact that the sample kurtosis is higher than expected with the exponential and GE distributions.

Computing initially the moment estimators under the SGE model we have the following estimates: $\widehat{\lambda}_M = 0.489$, $\widehat{\alpha}_M = 1.182$ and $\widehat{q}_M = 2.706$. Using the moment estimators as initial values, the maximum likelihood estimates are computed and presented in Table 4, for models GE and SGE jointly with the values for the AIC and BIC criteria.

Figure 4 depicts plots of the fitted GE and SGE models using the maximum likelihood estimates. Notice that model the fitted SGE model present heavier tails than the GE model. The QQ-plots for both models are presented in Figure 5. When SGE and GE models are compared using the LR statistics, the value of the corresponding statistic is 7.472 (p-value <0.004), which allows us to establish that the SGE model is more appropriate than the GE model for this data.

5.2 Illustration 2

The second illustration represents the remission times (in months) of a random sample of 128 bladder cancer patients (Lee and Wang, 2003). The observed data set is as follows: 0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06,7.09, 9.22, 13.80, 25.74, 0.50, 2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62,3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66,1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25,17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13,1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 21.73, 2.07, 3.36, 6.93, 8.65,12.63, 22.69.

Table 5 shows descriptive statistics corresponding to the remission times. Notice that sample kurtosis is high.

Based on the sample above, moment estimators for the parameters of the SGE model are $\widehat{\lambda}_M = 0.134$, $\widehat{\alpha}_M = 0.95$ and $\widehat{q}_M = 4.333$. Using these estimates as initial values, maximum likelihood estimates were computed for models SGE and GE and are reported in Table 6. Also shown are the estimated likelihood and AIC and BIC values for each model.

Figure 6 shows plots of estimated densities for models GE and SGE, with the latter model presenting heavier tails. QQ-plots for both models are presented in Figure 7. When SGE and GE models are compared by the LR test, the value of the corresponding statistic is 6.136 (p-value<0.007), which allows us to establish that the SGE model is more appropriate than the GE model for these data.

6 Concluding Remarks

In this paper we introduced a new extension of the generalized exponential (GE) distribution with a more flexible kurtosis coefficient. It is defined as the quotient between a GE random variable and the power of a uniform random variable. We call this distribution as the slashed generalized exponential (SGE) distribution. The GE model is a particular case. Real data illustrations indicate good performance of the proposed model.

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Acknowledgments

The authors acknowledge helpful comments and suggestions by the referee which substantially improved the presentation. The research of J. M. Astorga was supported by DIUDA 221183 (Chile). The research of H. W. Gómez was supported by SEMILLERO UA-2014 (Chile). The research of H. Bolfarine was supported by CNPq and Fapesp (Brasil).

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Table 1: Asymmetry and kurtosis coefficients for several parameter values of the SGE distribution.

α	q	$\sqrt{\beta}_1$	β_2
1	5	2.7186	21.9799
2		2.4079	20.0177
3		2.3344	20.1550
1	6	2.3896	14.0118
2		2.0402	11.8975
3		1.9317	11.4656
1	7	2.2456	11.7411
2		1.8794	9.6518
3		1.7553	9.1002
1	10	2.0945	9.8986
2		1.7119	7.8890
3		1.5724	7.2808

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Table 2: Empirical means and SD for the MLE estimators of λ, α and q.

			$n = 50$		
λ	α	q	$\widehat{\lambda}$ (SD)	$\widehat{\alpha}$ (SD)	$\widehat{q}$ (SD)
3.0	3.0	1.0	3.2621 (1.4544)	3.3869 (1.4836)	1.1197 (0.3880)
		2.0	3.2747 (1.6440)	3.3507 (1.3115)	2.3147 (1.2725)
		3.0	3.0658 (1.1807)	3.5043 (1.7394)	3.5512 (1.6306)
3.0	4.0	1.0	3.1889 (1.3702)	4.6917 (2.8133)	1.1430 (0.5340)
	5.0		3.1375 (1.2163)	5.8613 (2.8017)	1.1193 (0.3069)
	6.0		3.2447 (1.5147)	6.8845 (3.7011)	1.1012 (0.2621)
4.0	3.0	1.0	4.2271 (1.8242)	3.9785 (1.8378)	1.2022 (0.5815)
5.0			5.3436 (2.2076)	3.7851 (2.1606)	1.1604 (0.8336)
6.0			6.3449 (2.6961)	3.8717 (2.1751)	1.1789 (0.6789)
			$n = 100$		
λ	α	q	$\widehat{\lambda}$ (SD)	$\widehat{\alpha}$ (SD)	$\widehat{q}$ (SD)
3.0	3.0	1.0	3.1061 (0.8544)	3.3524 (1.2755)	1.0520 (0.1779)
		2.0	3.0693 (0.8022)	3.2605 (0.9821)	2.2905 (1.0257)
		3.0	3.0446 (0.7568)	3.2231 (0.8709)	3.4748 (1.5832)
3.0	4.0	1.0	3.1072 (0.8953)	4.6452 (2.5287)	1.0452 (0.1636)
	5.0		3.0126 (0.7916)	5.6185 (2.7968)	1.0631 (0.1651)
	6.0		3.0114 (0.7476)	6.7685 (3.6749)	1.0559 (0.1527)
4.0	3.0	1.0	4.0459 (1.1929)	3.2764 (1.4254)	1.0622 (0.1806)
5.0			5.0910 (1.4557)	3.3379 (1.2697)	1.0648 (0.1789)
6.0			6.1330 (1.7665)	3.3478 (1.5325)	1.0658 (0.1745)
			$n = 200$		
λ	α	q	$\widehat{\lambda}$ (SD)	$\widehat{\alpha}$ (SD)	$\widehat{q}$ (SD)
3.0	3.0	1.0	3.0436 (0.6302)	3.1569 (0.7862)	1.0298 (0.1272)
		2.0	3.0525 (0.5174)	3.1254 (0.5796)	2.1227 (0.5692)
		3.0	2.9961 (0.3555)	3.0450 (0.3603)	3.2773 (0.9974)
3.0	4.0	1.0	2.9705 (0.5894)	4.1315 (1.1579)	1.0326 (0.1154)
	5.0		3.0124 (0.5890)	5.3431 (1.7450)	1.0340 (0.1071)
	6.0		2.9697 (0.5571)	6.2757 (1.9706)	1.0352 (0.1126)
4.0	3.0	1.0	3.9919 (0.8222)	3.0871 (0.7677)	1.0321 (0.1155)
5.0			4.9758 (1.0249)	3.0782 (0.7568)	1.0345 (0.1217)
6.0			6.0281 (1.1897)	3.1224 (0.7308)	1.0288 (0.1227)

Table 3: Descriptive statistics for the repair times.

n	$\overline{X}$	s^2	b_1	b_2
46	3.607	24.445	2.888	11.803

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Table 4: Maximum likelihood estimates for models GE and SGE and AIC and BIC values for the repair times.

Parameters	GE (SD)	SGE (SD)
$\widehat{\lambda}$	0.269 (0.054)	1.523 (1.212)
$\widehat{\alpha}$	0.958 (0.190)	2.238 (1.457)
$\widehat{q}$	-	1.138 (0.479)
Loglikelihood	-104.983	-101.247
AIC	213.966	208.494
BIC	217.623	213.980

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Table 5: Descriptive statistics for remission data.

n	$\overline{X}$	s^2	b_1	b_2
128	9.366	110.425	3.287	18.483

Table 6: Maximum likelihood estimates, likelihood, AIC and BIC values for the remission data.

Parameters	GE (SD)	SGE (SD)
$\widehat{\lambda}$	0.121 (0.014)	0.231 (0.053)
$\widehat{\alpha}$	1.218 (0.149)	1.511 (0.240)
$\widehat{q}$	-	2.416 (0.751)
Loglikelihood	-413.078	-410.010
AIC	830.155	826.019
BIC	835.859	834.575

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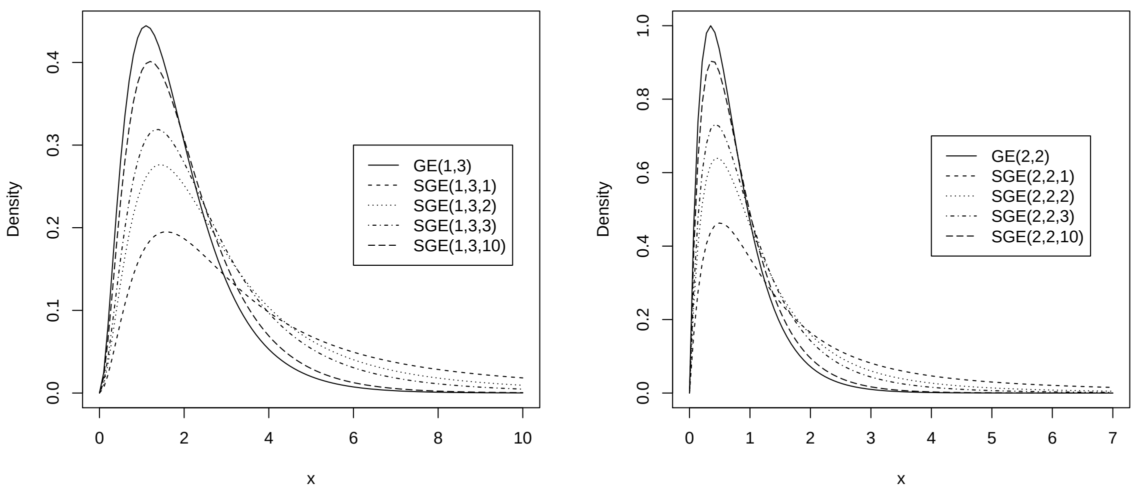

Figure 1: Plot of the Slashed Generalized Exponential density, $SGE(\lambda, \alpha, q)$.

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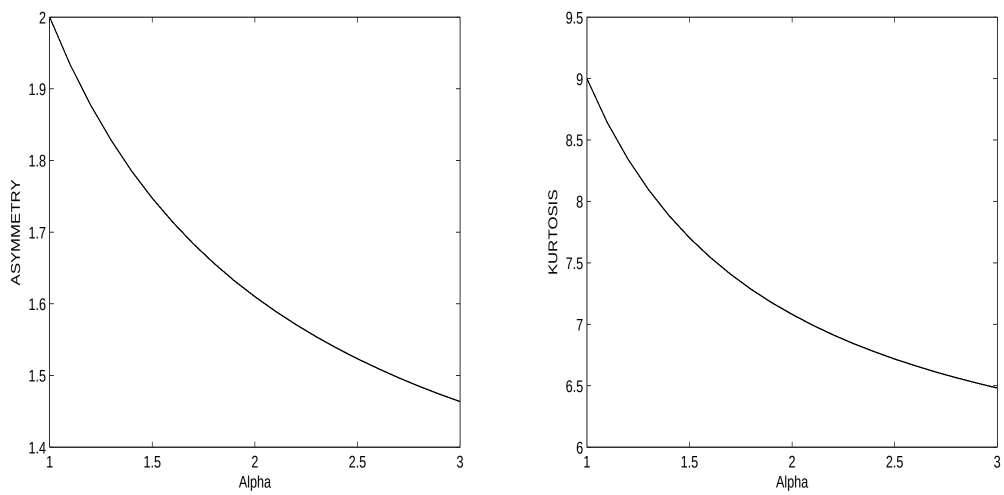

Figure 2: Plots of the generalized exponential's asymmetry and kurtosis.

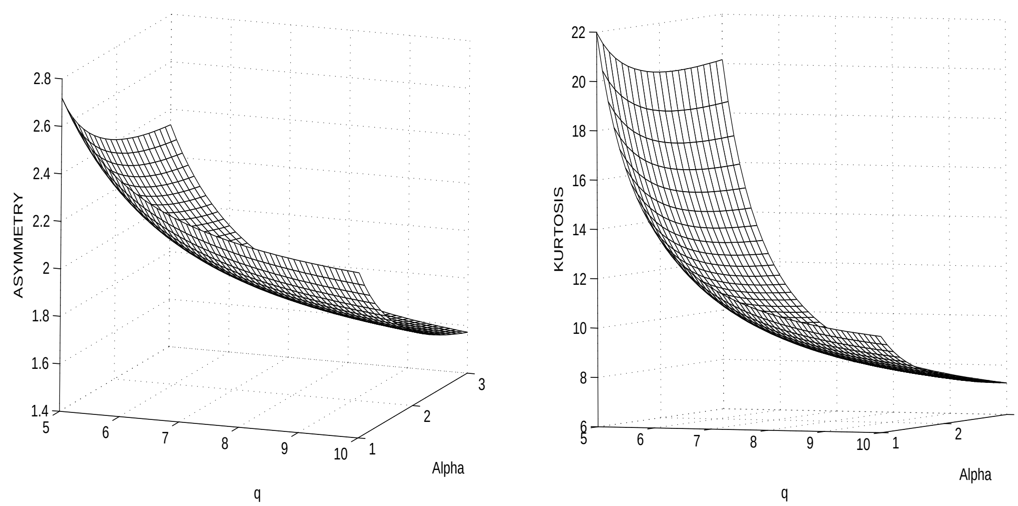

Figure 3: Plots of the slashed generalized exponential's asymmetry and kurtosis.

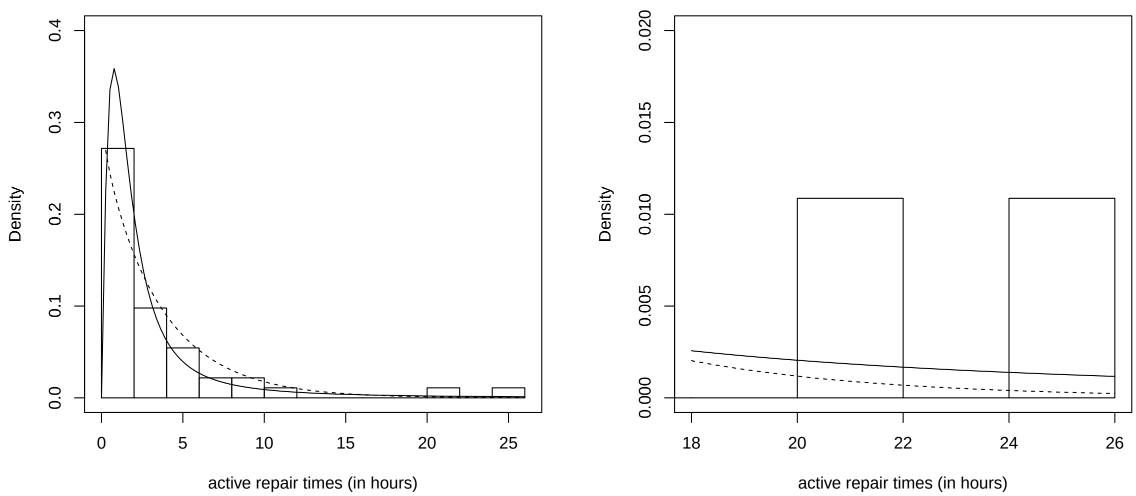

Figure 4: Left panel: models fitted by the maximum likelihood approach for active repair time data set: SGE (solid line) and GE(dashed line). Right panel: Plots of the tails for both models

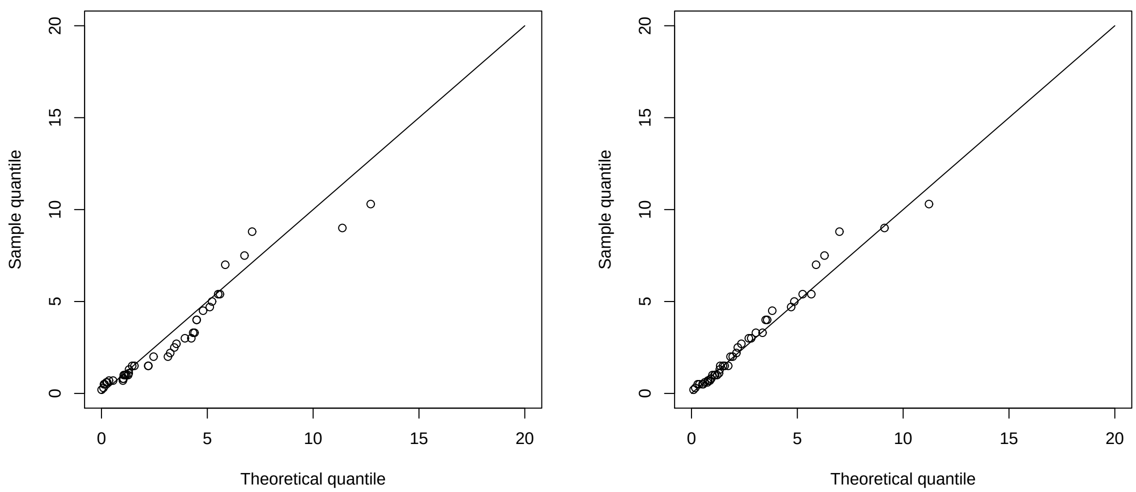

Figure 5: Left panel: QQ-plot for model GE. Right panel: QQ-plot for model SGE

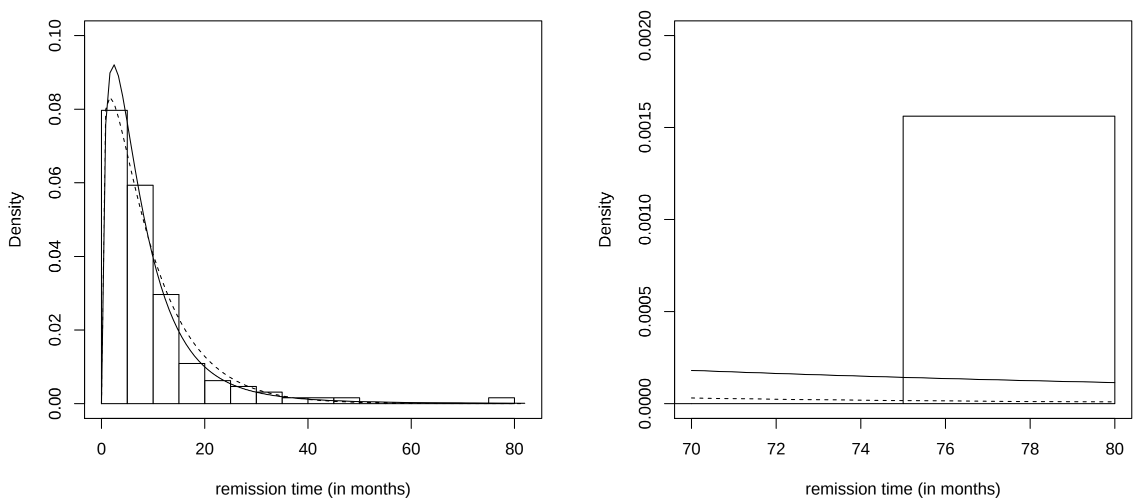

Figure 6: Left panel: models fitted by maximum likelihood method for remission times: SGE (solid line) and GE(dashed line). Right panel: part of right tails for both model.

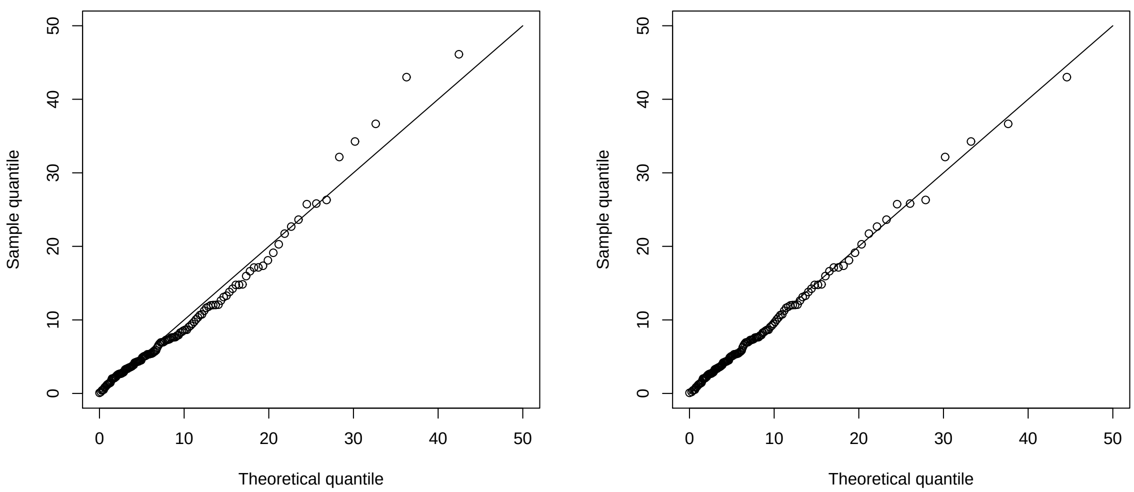

Figure 7: Left Panel: QQ-plot for the GE model. Right panel: QQ-plot for model SGE

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